7. Surface

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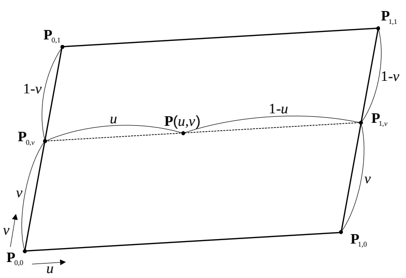
Bilinear Surface

- A bilinear surface is derived by interpolating the four data points with the linear equations in the parameters *u* and *v* such that the resulting surface has the four points at its corners

$$\mathbf{P}_{0,v} = (1-v)\mathbf{P}_{0,0} + v\mathbf{P}_{0,1}$$

$$\mathbf{P}_{1,v} = (1-v)\mathbf{P}_{1,0} + v\mathbf{P}_{1,1}$$

$$\mathbf{P}(u,v) = (1-u)\mathbf{P}_{0,v} + u\mathbf{P}_{1,v}$$

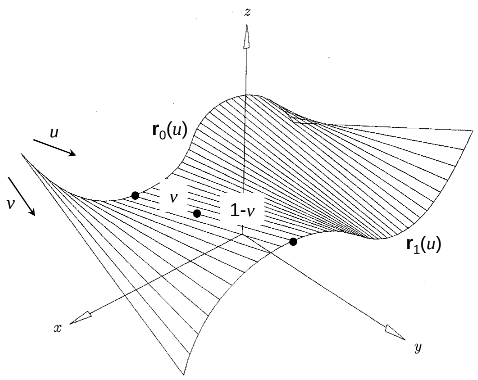


$$\mathbf{P}(u,v) = (1-u)\left[(1-v)\mathbf{P}_{0,0} + v\mathbf{P}_{0,1}\right] + u\left[(1-v)\mathbf{P}_{1,0} + v\mathbf{P}_{1,1}\right]$$

$$= \begin{bmatrix}(1-u)(1-v) & u(1-v) & (1-u)v & uv\end{bmatrix}\begin{bmatrix}\mathbf{P}_{0,0}\\ \mathbf{P}_{1,0}\\ \mathbf{P}_{0,1}\\ \mathbf{P}_{1,1}\end{bmatrix} \quad \begin{matrix}0 \le u \le 1\\ 0 \le v \le 1\end{matrix}$$

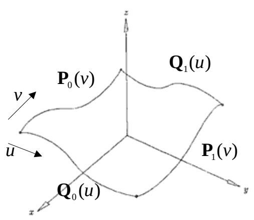
Ruled Surface

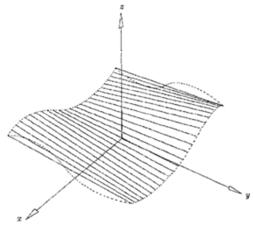
$$\mathbf{r}(u,v) = (1-v)\mathbf{r}_0(u) + v\mathbf{r}_1(u), \qquad 0 \le v \le 1$$



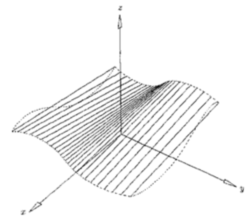
0-3

Coon's Patch

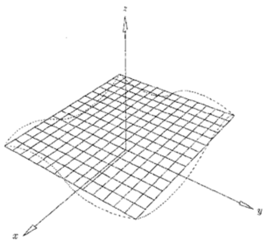


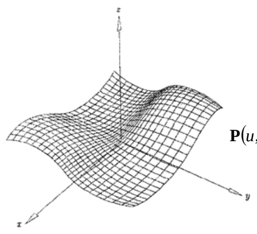


$\mathbf{P}_1(u,v) = (1-u)\mathbf{P}_0(v) + u\mathbf{P}_1(v)$



$\mathbf{P}_2(u,v) = (1-v)\mathbf{Q}_0(u) + v\mathbf{Q}_1(u)$





$$\begin{aligned}\mathbf{P}(u,v) &= \mathbf{P}_1(u,v) + \mathbf{P}_2(u,v) - \mathbf{P}_3(u,v)\\ &= (1-u)\mathbf{P}_0(v) + u\mathbf{P}_1(v) + (1-v)\mathbf{Q}_0(u) + v\mathbf{Q}_1(u)\\ &\quad -(1-u)(1-v)\mathbf{P}_{0,0} - u(1-v)\mathbf{P}_{1,0} - (1-u)v\mathbf{P}_{0,1} - uv\mathbf{P}_{1,1}\end{aligned}$$

$0 \le u \le 1, 0 \le v \le 1$

$\mathbf{P}_3(u,v) = (1-u)(1-v)\mathbf{P}_{0,0} + u(1-v)\mathbf{P}_{1,0} + (1-u)v\mathbf{P}_{0,1} + uv\mathbf{P}_{1,1}$

0-4

Bicubic Patch

- Bicubic Patch
  - Extension of the parametric cubic curve formulation
    - Boundary curves are parametric cubics or Hermites
    - The Interior is defined by blending functions
- Algebraic Form

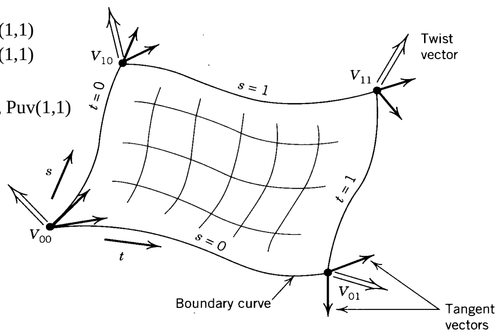
$$\mathbf{P}(u,v)=\sum_{i=0}^{3}\sum_{j=0}^{3}\mathbf{a}_{ij}u^i v^j \qquad 0\le u\le 1, 0\le v\le 1$$

– Matrix form

$$\mathbf{P}(u,v)=\begin{bmatrix}1 & u & u^2 & u^3\end{bmatrix}\begin{bmatrix}\mathbf{a}_{00} & \mathbf{a}_{01} & \mathbf{a}_{02} & \mathbf{a}_{03}\\ \mathbf{a}_{10} & \mathbf{a}_{11} & \mathbf{a}_{12} & \mathbf{a}_{13}\\ \mathbf{a}_{20} & \mathbf{a}_{21} & \mathbf{a}_{22} & \mathbf{a}_{23}\\ \mathbf{a}_{30} & \mathbf{a}_{31} & \mathbf{a}_{32} & \mathbf{a}_{33}\end{bmatrix}\begin{bmatrix}1\\ v\\ v^2\\ v^3\end{bmatrix}$$

Bicubic Patch (2)

- Geometric Form
  - 16 vector equations are required for 16 algebraic coefficient vectors
  - 16 Boundary conditions
    - 4 corner points:
      - P(0,0), P(0,1), P(1,0), P(1,1)
    - 8 tangent vectors at corner points:
      - Pu(0,0), Pu(0,1), Pu(1,0), Pu(1,1)
      - Pv(0,0), Pv(0,1), Pv(1,0), Pv(1,1)
    - 4 twist vectors at corner points
      - Puv(0,0), Puv(0,1), Puv(1,0), Puv(1,1)



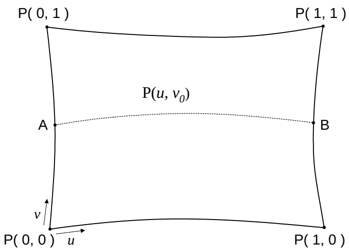
Cubic Patch (3)

$$\mathbf{P}(u,v)=\begin{bmatrix}F_1(u) & F_2(u) & F_3(u) & F_4(u)\end{bmatrix}\begin{bmatrix}\mathbf{P}(0,0) & \mathbf{P}(0,1) & \mathbf{P}_v(0,0) & \mathbf{P}_v(0,1)\\ \mathbf{P}(1,0) & \mathbf{P}(1,1) & \mathbf{P}_v(1,0) & \mathbf{P}_v(1,1)\\ \mathbf{P}_u(0,0) & \mathbf{P}_u(0,1) & \mathbf{P}_{uv}(0,0) & \mathbf{P}_{uv}(0,1)\\ \mathbf{P}_u(1,0) & \mathbf{P}_u(1,1) & \mathbf{P}_{uv}(1,0) & \mathbf{P}_{uv}(1,1)\end{bmatrix}\begin{bmatrix}F_1(v)\\ F_2(v)\\ F_3(v)\\ F_4(v)\end{bmatrix}$$

$$0 \le u \le 1,\ 0 \le v \le 1$$

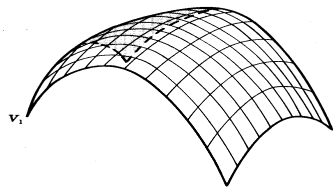
- Blending Functions (Hermite Curve Eq'ns)

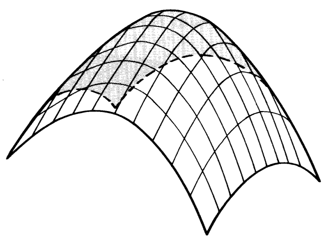
$$F_1(u)=1-3u^2+2u^3$$
$$F_2(u)=3u^2-2u^3$$
$$F_3(u)=u-2u^2+u^3$$
$$F_4(u)=-u^2+u^3$$



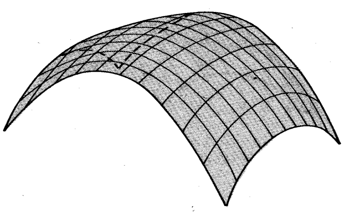
Cubic Patch (4)

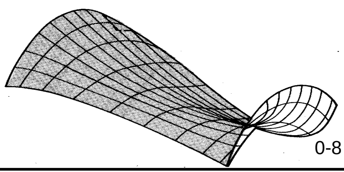
- Effect of Variation in Twist Vector





- Effect of Variation in Tangent Vector





Ferguson Patch (F-Patch)

- Definition:
  - Setting all twist vector to zero

    $\mathbf{P}_{uv}(0,0) = \mathbf{P}_{uv}(0,1) = \mathbf{P}_{uv}(1,0) = \mathbf{P}_{uv}(1,1) = 0$
  - Not commonly used in practice because they force the surface to flatten at the corners
- Disadvantage:
  - No intuitive feel for the values of the tangent and twist vectors is available to the user

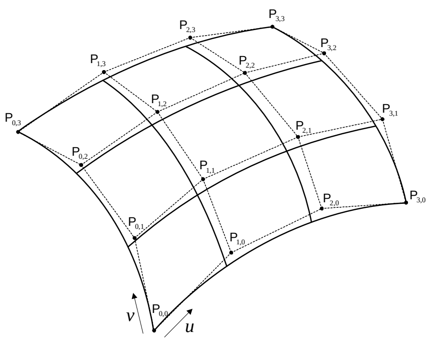
Bezier Surface

$$\mathbf{P}(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} \mathbf{P}_{i,j} B_{i,n}(u) B_{j,m}(v) \qquad 0 \le u \le 1,\ 0 \le v \le 1$$

$P_{i,j}$: control vertices

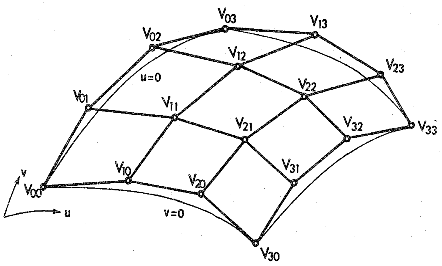
$B_{i,n}(u)$, $B_{j,m}(v)$: Bernstein blending functions in the u and v directions

● Note: **n** does not have to be the same as **m**



Bicubic Bezier Surface

$$\mathbf{P}(u,v) = \sum_{i=0}^{3}\sum_{j=0}^{3} B_{i,3}(u)B_{j,3}(v)\mathbf{V_{ij}}$$
$$= \sum_{i=0}^{3} B_{i,3}(u)\left(\sum_{j=0}^{3} B_{j,3}(v)\mathbf{V}_{ij}\right)$$
$$= \sum_{i=0}^{3} B_{i,3}(u)\mathbf{b_i}(v)$$
$$= \mathbf{UMBM^TV^T}$$



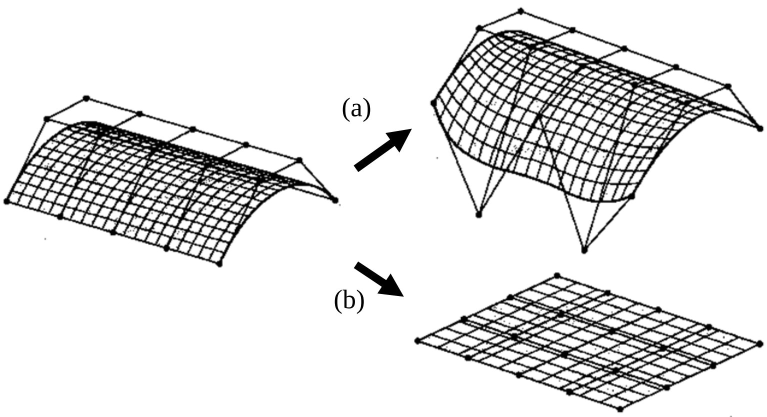
여기서,

$$U = \begin{bmatrix}1 & u & u^2 & u^3\end{bmatrix},\ V = \begin{bmatrix}1 & v & v^2 & v^3\end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ 1 & 3 & -3 & 1 \end{bmatrix},\ B = \begin{bmatrix} V_{00} & V_{01} & V_{02} & V_{03} \\ V_{10} & V_{11} & V_{12} & V_{13} \\ V_{20} & V_{21} & V_{22} & V_{23} \\ V_{30} & V_{31} & V_{32} & V_{33} \end{bmatrix}$$

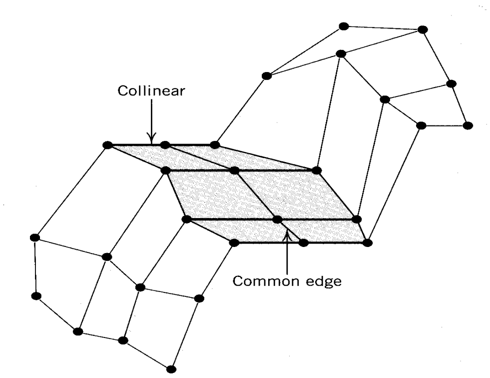
Bezier Surface

- Effect of Moving Control Points
  - (a) At the boundary curves
  - (b) On the interior part of the surface



Bezier Surface

- Continuity of Bicubic Bezier Surfaces
  - First degree parametric continuity is enforced along the common edge between two patches



0-13

B-Spline Surface

$$P(u,v)=\sum_{i=0}^{n}\sum_{j=0}^{m}\mathbf{V}_{i,j}\ N_{i,k}(u)N_{j,l}(v) \qquad s_{k-1}\le u\le s_{n+1},\ t_{l-1}\le v\le t_{m+1}$$

$V_{i,j}$: control points

$N_{i,k}(s)$, $N_{j,l}(t)$: B-spline blending functions in the u and v directions

- Note: ***k*** does not have to be the same as ***l***

0-14

Bicubic B-spline Surface (Uniform)

$$\mathbf{P}(u,v) = \sum_{i=0}^{3}\sum_{j=0}^{3} N_{i,4}(u) N_{j,4}(v) \mathbf{V}_{ij}$$

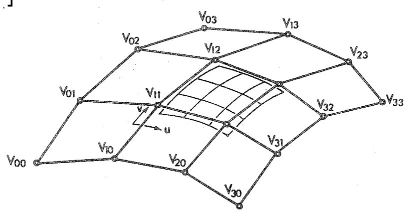
$$= \mathbf{UNBN^T V^T} \; with \;\; 0 \le u,v \le 1$$

여기서,

$$U = \begin{bmatrix} 1 & u & u^2 & u^3 \end{bmatrix}; \quad V = \begin{bmatrix} 1 & v & v^2 & v^3 \end{bmatrix}$$

$$B = \begin{bmatrix} V_{00} & V_{01} & V_{02} & V_{03} \\ V_{10} & V_{11} & V_{12} & V_{13} \\ V_{20} & V_{21} & V_{22} & V_{23} \\ V_{30} & V_{31} & V_{32} & V_{33} \end{bmatrix}$$

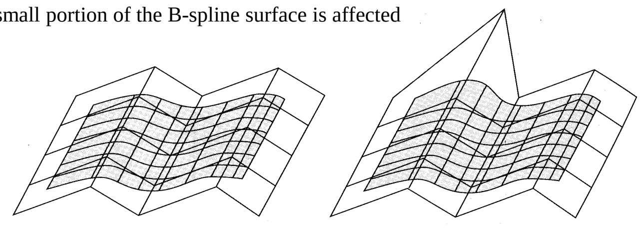
$$N = \frac{1}{6}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -3 & 3 & -2 & -1 \\ 2 & -2 & 1 & 1 \end{bmatrix}$$



B-Spline Surface

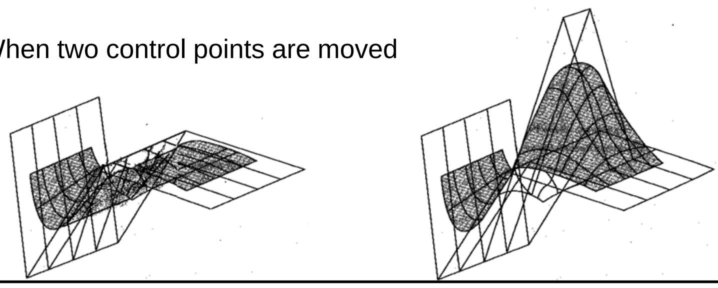
– When one control point is moved

only a small portion of the B-spline surface is affected



(a) (b)

– When two control points are moved



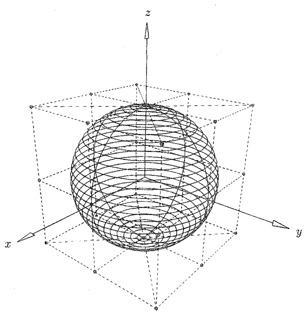
NURBS Surface

$$P(u,v)=\frac{\sum_{i=0}^{n}\sum_{j=0}^{m} h_{i,j}\,\mathbf{V}_{i,j}\,N_{i,k}(u)\,N_{j,l}(v)}{\sum_{i=0}^{n}\sum_{j=0}^{m} h_{i,j}\,N_{i,k}(u)\,N_{j,l}(v)}$$

$$s_{k-1} \le u \le s_{n+1}$$
$$t_{l-1} \le v \le t_{m+1}$$

$\mathbf{V}_{i,j}$: x, y, and z coordinates of the control points
$h_{i,j}$: homogeneous coordinates of the control points

- Quadric (Quadratic) NURBS Surface로 Cylinder, Cone, Sphere, Paraboloid, Hyperboloid를 정확히 나타낼 수 있다.

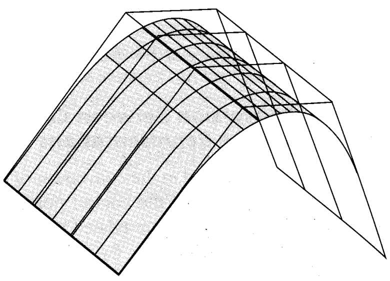


0-17

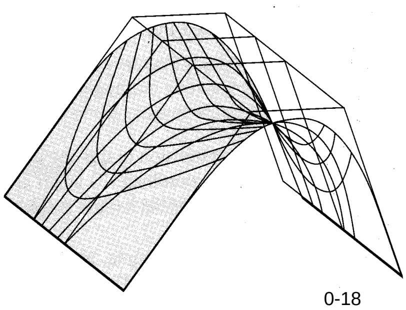
NURBS Surface

- Effect of Weights
  - The weights provide an additional degree of freedom for the shape of surface

– Larger values of weights at the interior control points



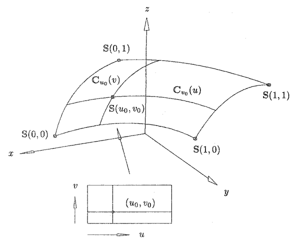
– Lower values of weights at the top interior control points



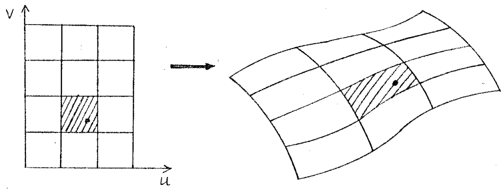
0-18

Patch, Composite, Trimmed Surface (1)

- Surface Patch:
  단일의 식으로 표현된 곡면



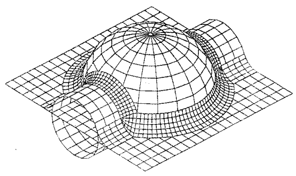
- Composite Surface :
  piecewisely defined continuous surface



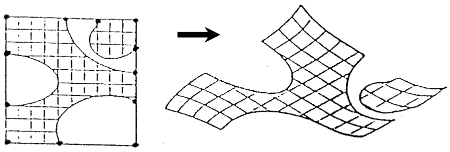
0-19

Patch, Composite, Trimmed Surface (2)

- Compound Surface: 위상학적 관계가 없는 곡면들의 단순 집합



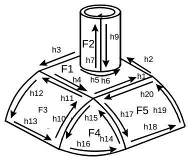
- Trimmed Surface (face): composite surface의 일부 영역

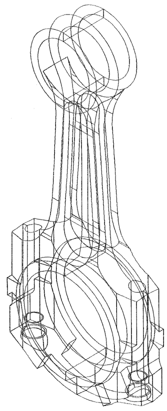


0-20

Patch, Composite, Trimmed Surface (3)

- Skin or Shell:
  Topological Relationship을 갖는 trimmed surface의 집합



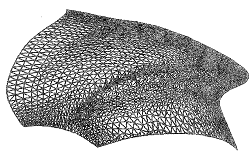


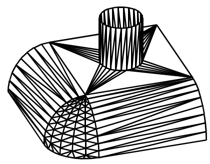
- Solid: 닫혀진 skin

0-21

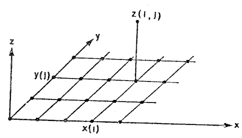
근사곡면 모델

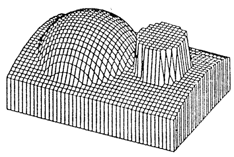
- Polyhedral Surface (facetted surface)





- Z-map Surface Model



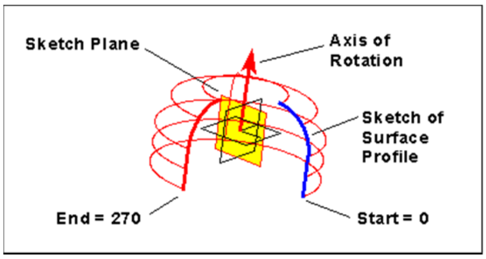


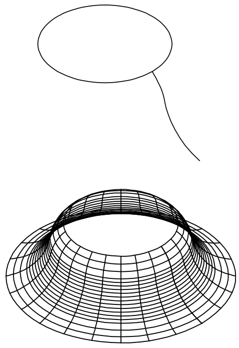
- 기타 : G-buffer Model, Discrete point Model,
  Polyhedral with normal vector at each vertex

⇨ Geometric processing 간단, 신속, robust, 정확성 낮음, memory 소요량 낮음

0-22

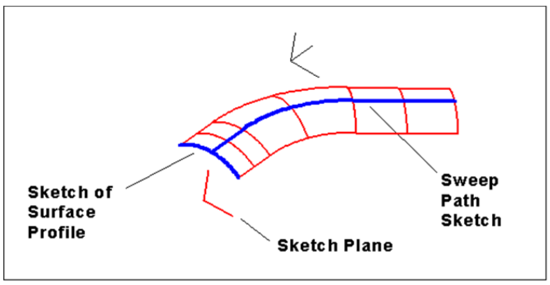
Rotational Sweep / Revolve

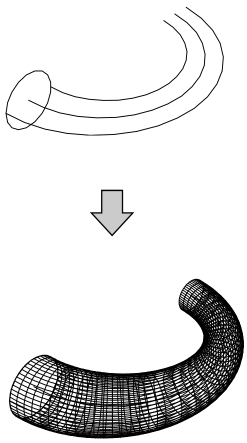




0-23

Sweep

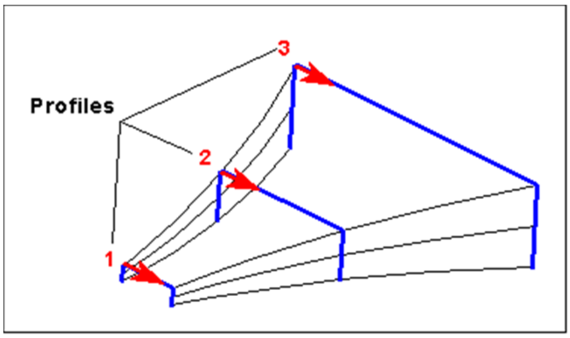




0-24

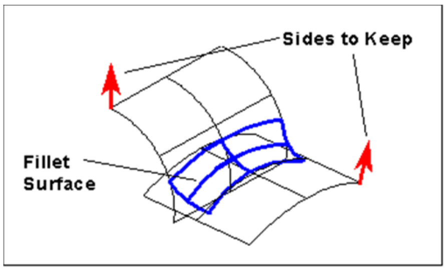
Loft

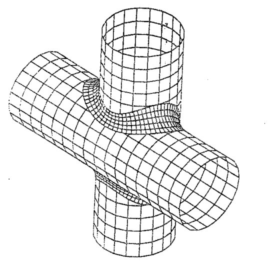
- 연속적인 단면을 포함하는 곡면



0-25

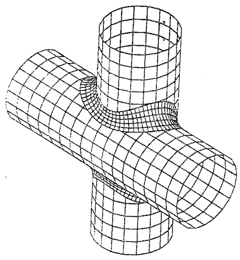
Fillet



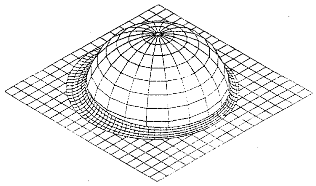


0-26

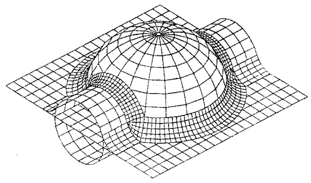
Blending



고정반경 블렌딩



가변모서리 블렌딩



코너 블렌딩

Trimming